# Probabilistic Context-free Grammars and Other Syntactic Language Models



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### Kelsey and other Grammers

- A grammar here is another word for a language model
- They consist of four sets G = (Σ, N, S, P) terminals – word types; lowest nodes in syntax trees Examples: dog, the, eats non-terminals – phrasal types; middle nodes in syntax trees
  - Examples: VP, DET, NP
  - start symbol "S"; the top node in syntax trees

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S	$\rightarrow$	NP VP
NP	$\rightarrow$	DET N
DET	$\rightarrow$	the
Ν	$\rightarrow$	cat
VP	$\rightarrow$	V PP
V	$\rightarrow$	sat
PP	$\rightarrow$	P NP
Ν	$\rightarrow$	mat

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- Then people started to think of these trees as the actual **structure** of a sentence
- Confusion ensued

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- CFGs can generate and recognize **center embedding**, but not more complex word order phenomena, so effectively CFG parse trees have **no crossing lines**
- Non-projective dependency grammars are more or less equivalent to CFGs (they have the same weak generative capacity)

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- Here is a link to a list of syntactic treebanks

#### PCFGs

- We can induce a **probabilistic context-free grammar** (PCFG) from the treebank
- With multiple annotated sentences, we can get probabilities for production rules. Example:

1.0	S	$\rightarrow$ NP VP
0.6	NP	$\rightarrow$ DET N
0.4	NP	ightarrow ADJ N
0.7	VP	$\rightarrow V NP$
0.3	VP	$\rightarrow V$
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 Notice that the probabilities for each left-hand side must sum to one (unity)

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- If we have a treebank, we can start with just counting how often productions occur (maximum likelihood estimation)
- If we don't have a treebank, we can still use unannotated text, and apply the **inside-outside algorithm**
- The inside-outside algorithm is just the expectation-maximization (EM) algorithm applied to trees
- We start by randomly initializing probabilities to all possible rule productions, then use EM to search for good rule probabilities that maximize the likelihood of the training set

#### Inside-Outside Algorithm

- The inside-outside algorithm uses inside- and outside-probabilities:
  - Inside probability:  $\beta_j(p,q) = P(w_{pq}|N_{pq}^j,G)$
  - Outside probability:  $\alpha_j(p,q) = P(w_{1(p-1)}, N_{pq}^j, w_{(q+1)m}|G)$



### String Probabilities

 We use the inside probability of the entire sentence to get the probability of that sentence:

$$P(w_{1\,m}|G) = P(N^1 \stackrel{*}{\Rightarrow} w_{1\,m}|G) = \beta(1,m)$$

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- Probabilities of ambiguous parses at a given non-terminal are summed, since either parse could have produced the final substring

PCFGs *vs. n*-gram Language Models (Lexicalized Probabilistic Regular Grammars)

- PCFGs can better handle long-distance dependencies like subject-verb agreement and filler-gap dependencies
- PCFGs usually give worse perplexity than *n*-gram LMs. Why?

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- PCFGs take longer to train
- PCFGs need manually-annotated treebanks to give decent results
- PCFG parsers (eg. CKY) are usually not incremental