Probabilities, probably

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- That is, whether a particular sentence was **grammatical** or **ungrammatical**
- This is, of course, an overly simplistic view of natural language
- This week, we're going to take a more subtle approach
- The formal languages & grammars are still relevant, but we're going to add **probabilities** to them

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- A **probability distribution** is the probabilities for all the possible outcomes

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- Ok, so maybe it's not a very good idea

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- It doesn't take into account a word's context

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- **Likelihood** is the probability of the entire data, given our model. The higher this is, the better it is at predicting the data...

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- Then they'll divide that number by the number of words in the data. Why?
- This gives us the average number of binary choices the model made to predict each outcome in the data, or **cross entropy**

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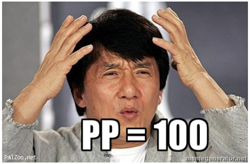

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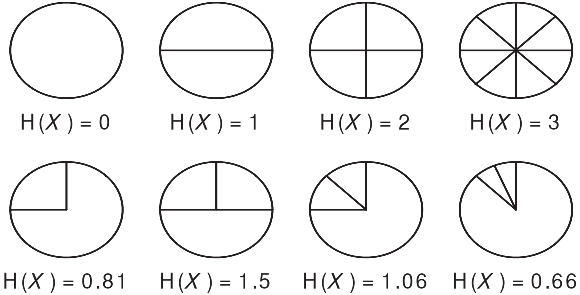

(courtesy of Koehn, 2010)

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- Why is this type of estimation called *maximum likelihood*?
- Are there ever any unseen events in language data?
- How could we handle unseen events (not seen before in the training set)?

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- Rules have an unsmoothed, uniform distribution – each rule has an equiprobable chance
- Grammatical: $p(x) > 0$, ungrammatical: $p(x) = 0$