Formal Models of Language

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Introduction

Hi!

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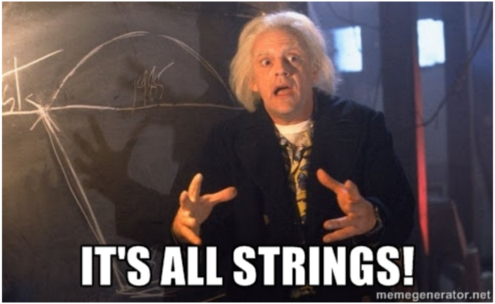

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- (Similar to musical/poetic form analysis)

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- If two different grammars can generate/accept the same structures as well, then they have the same **strong generative capacity**

Formal Language Hierarchy

	Formal Language
	Non-Turing-acceptable
0:	**Recursively enumerable**
	Recursive/ Decidable
1:	Context-sensitive
	Indexed
	Mildly context-sensitive
2:	**Context-free**
	Deterministic context-free
3:	**Regular**
	Finite

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This is extended from the older *Chomsky hierarchy*. We'll discuss the ones in boldface, as they're relevant to natural languages.

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- For example: long-distance dependencies, complex reordering in machine translation, reduplication, etc.
- You can also get an idea of how fast or slow it will take for a computer (or human) to process sequential stuff (like natural language!)

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- For natural language, this would correspond to having a finite number of possible sentences

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- (There's more discussion on the interwebs if you're interested)

Regular Languages

- Ok, so maybe for now it's too difficult to list all possible sentences
- Let's assume that the vocabulary (Σ) is still fixed (or finite), but we can generate an infinite number of sentences from this fixed vocab
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- Processing regular languages can be done in linear time ($\mathcal{O}(n)$), with a constant size of memory ($\mathcal{O}(1)$)

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Mildly Context-Sensitive Languages

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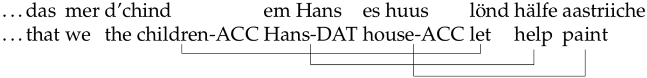

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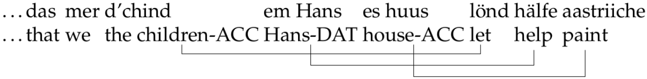

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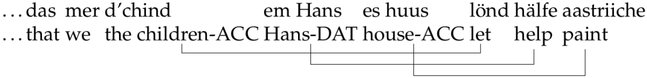

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- Processing MCS languages can be done in about $\mathcal{O}(n^6)$ time, with quadratic memory usage ($\mathcal{O}(n^2)$)
- Mildly context-sensitive is very different from context-sensitive, which is much more powerful
- Some grammar formalisms that can handle MCS langs:
    - Tree Adjoining Grammar (TAG)
    - Combinatory Categorial Grammar (CCG)
    - Linear Indexed Grammars (LIG) (easy to understand)
    - Head Grammars (HG)

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- Note that these grammar formalisms can place some restrictions on word order, but they still accept/generate recursively enumerable languages. How is that so? Additional grammar rules can work around such restrictions to accept/generate the string.